# ASTR121 Lab 5 HI Rotation Curve of the Milky Way

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**Abstract**

Keplerian orbital mechanics tell us that the mass contained within a certain radius within a spiral galaxy can be derived from the orbital velocity at that radius. We can derive rotational velocities at incrementing radii by observing Doppler shift of 21 cm radiation emitted from neutral hydrogen clouds orbiting within the galactic disk. We derived the Doppler shifts, and thus radial velocity, of these clouds from luminosities over wavelengths gained from observing at several galactic longitudes, and separated individual spectra over the range of line of sight, choosing the clouds with fully observed tangential velocity as indicative of orbital velocities at their orbital radius within the galactic disk. Performing linear fit over the relationship between derived orbital radii and derived orbital velocities of the observational data, we found that a radius of 4 kpc from the galactic center contains $(1.0425 \times 10^{41} \pm 4.7779 \times 10^{9})$ kg, or $(5.2412 \times 10^{10} \pm 2.4022 \times 10^{-21})$ $M_{\odot}$.

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# 1 Introduction

## 1.1 Neutral hydrogen

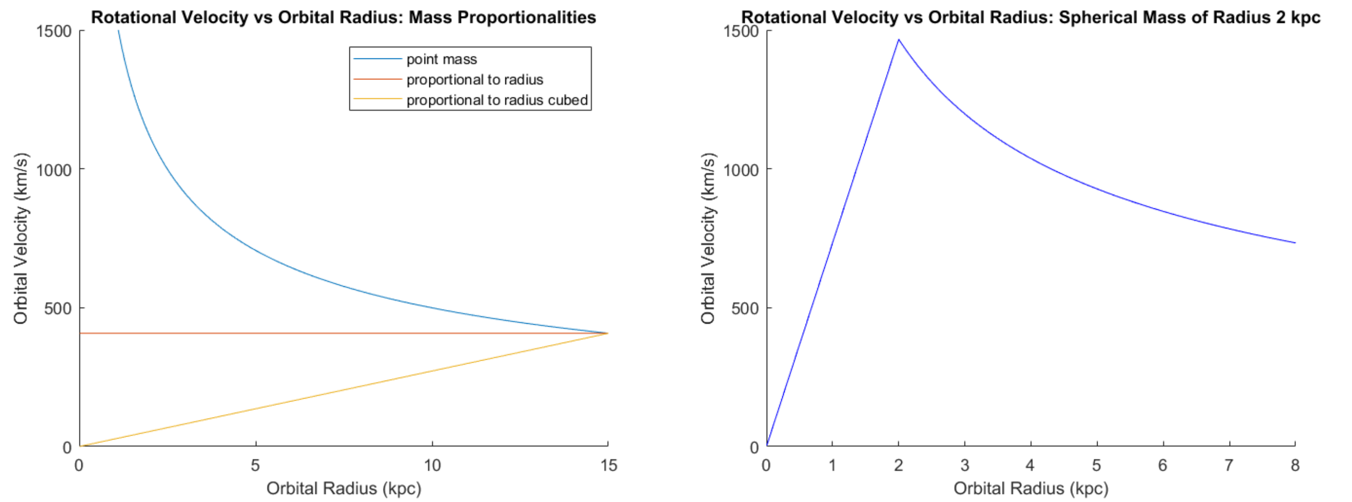
Neutral hydrogen (HI) clouds permeate interstellar space, and exist throughout the galactic disk of the Milky Way. They consist of atomic hydrogen, and emit radiation around 21 cm wavelengths in response to energy changes as spin states orient into parallel configurations between the proton and electron of the neutral hydrogen atom. This emission penetrates interstellar dust significantly farther than emission at optical wavelengths, and thus is suitable for studying the structure of spiral galaxies.

## 1.2 Rotation curves

Kepler's laws of orbital motion tell us that the velocity of an object traveling in a circular orbit is wholly dependent upon the mass contained within its orbit, as shown here,

$$v = \sqrt{\frac{G \times m}{r}} \tag{1}$$

where $v$ is velocity, $r$ is radius, $m$ is mass within that radius, and $G$ is a constant equal to $6.674\,08 \times 10^{-11}\,\mathrm{m^2\,kg^{-1}\,s^{-2}}$ [5]. Using this formula, we modeled several consistent mass distributions (Figure 1a), as well as an inconsistent mass distribution comprised of a spherical mass with a radius of 2 kpc (Figure 1b), to plot orbital velocity as a function of distance from the central point, akin to orbital radius.




(a) Orbital velocity vs orbital radius for three modeled mass proportions of a consistent nature

(b) Orbital velocity vs orbital radius for a galaxy with mass contained entirely within a 2 kpc sphere

Figure 1: Theoretical rotational velocity curves for several mass distributions

As can be seen in Figure 1a, varying the distribution of mass as a proportion of orbital radius will change the shape of the rotation curve before the full galactic radius, but will converge to the same orbital velocity once at the galactic radius, as the same amount of mass is contained within the total galaxy in every case.

## 1.3 Emission and galactic rotation

Orbital velocity can thus infer mass contained within a certain radius within a galaxy. We can obtain orbital velocities by observing clouds neutral hydrogen, and measuring the redshift occurring between the observed radiation and the known frequency of emission from neutral hydrogen clouds, 1420.405 752 MHz [4].

Ultimately, this means that mass distribution within a galaxy can be determined by measuring the velocity of material orbiting at incrementing radii, by observing clouds of neutral hydrogen along several galactic longitudes left of the galactic center (and so all moving with positive radial velocity).

As there may be several sources of HI emission within the line of sight of the observation, the wavelength of interest in this calculation is that with the largest redshift in the measurement, corresponding to the highest positive radial velocity and thus the radius tangential to the angle of observation.

# 2 Data and Methodology

In this lab, we first modeled galactic velocity curves (Figure 1), then examined luminosity data over a range of frequencies obtained from line of sight observations at specific galactic longitudes, at segmented intervals measuring 4° (Figure 2), in order to obtain rotational velocity from the spectral data.

## 2.1 Obtaining radial velocity from left-hand start of signal

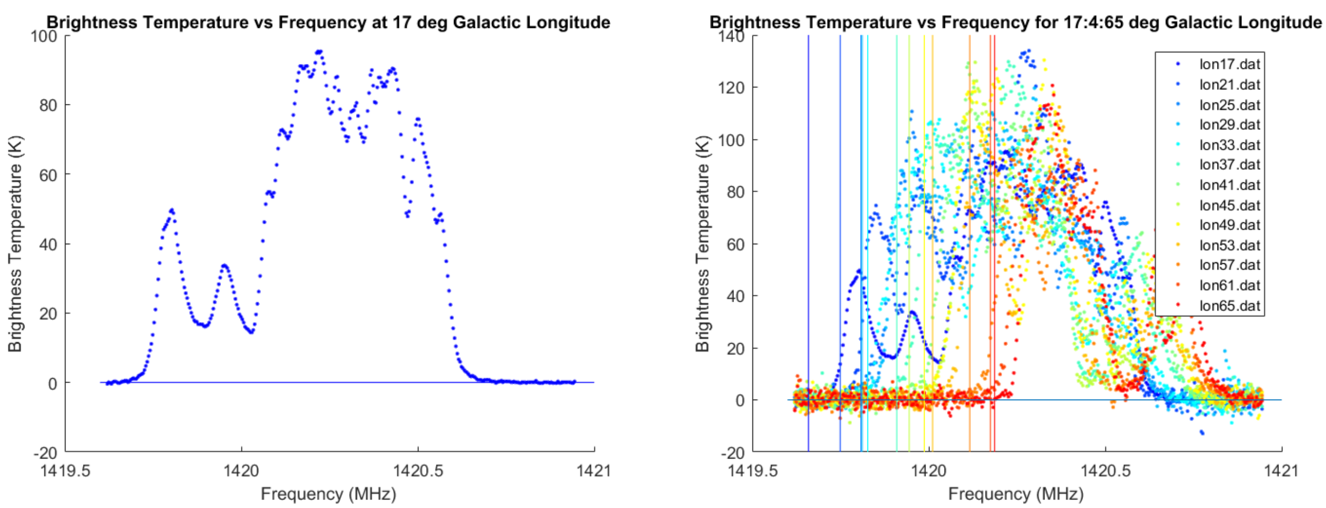
The data used in this lab consisted of observations of brightness temperatures through a range of frequencies made over several positive galactic longitudes, ensuring consistently redshifted wavelengths and thus positive radial velocities. The most important of these is the cloud with fully observed radial motion to the line of sight of the observation, as this is the only cloud for which Doppler shift will reveal the true tangential velocity of the cloud.

The cloud with fully observed tangential velocity in this data is the one with highest observed redshift from the emitted wavelength, which in this case is the signal of lowest frequency. Determining the left-hand start of signal from noise will thus determine the frequency of this cloud, and consequentially the redshift.

Assuming the speed of light $c$ is exactly $299\,792\,458\,\mathrm{m\,s^{-1}}$ [5], the radial velocity $v_r$ of an object emitting light at frequency $f_{emit}$ (which in the case of HI is $1420.405\,752\,\mathrm{MHz}$ [4]) being observed at frequency $f_{obs}$ is derived using the following method from the lab handout [1],

$$v_r = \frac{(f_{emit} - f_{obs})}{f_{obs}} \times c \tag{2}$$

We can then make the assumption that orbital velocity obtained in this matter can represent the orbital velocity of all material orbiting at the same radius within the galactic disk, and use that and Kepler's formula to infer the mass contained within each incremental radius, as observational galactic longitude increases. Data at galactic longitude $(17 \pm 1)°$ is shown in Figure 2a over a horizontal zero line.




(a) Spectral data at $(17 \pm 1)°$ galactic longitude

(b) Spectral data of 12 galactic longitudes between $(17 \pm 1)°$ through $(65 \pm 1)°$, in segmented increments of 4°, with respective likely left-hand start of signal frequencies indicated by colored vertical lines

Figure 2: Spectral data from line of sight observations at 12 galactic longitudes

By representing noise as fluctuations of brightness temperature over zero, we were able to approximate the left-hand start of signal frequency mathematically as the frequency following the frequency corresponding to the last brightness temperature lower than the average of brightness temperature extrema of the values between the first value and the last negative brightness temperature before $1420.405\,752\,\mathrm{MHz}$ [4]. In other words, the frequency indicating the left-hand start of signal corresponds to the last negative brightness temperature before the region of definite presence of signal, when adjusted for noise by subtracting the mean of the highest amplitude variations before the region of definite presence of signal, plus one index. Figure 2b shows the likely left-hand start of signal frequencies derived in this manner for every observed galactic longitude.

This method returns a frequency with symmetric uncertainty equal to the frequency difference between the returned frequency and the next highest frequency corresponding to a negative brightness temperature. This uncertainty is justified as the interval length of possible values between the region of definite lack of signal and the region of definite presence of signal.

### 2.2 Obtaining orbital velocity, radius, and mass from fully tangential radial velocity

To obtain true tangential orbital velocity $v_{orbital}$ of an object with observed radial velocity $v_r$ at galactic longitude $l_{object}$, we needed to add the angled component of solar motion in the Milky Way, $v_\odot = (299 \pm 15)\,\mathrm{km\,s^{-1}}$ towards galactic longitude $l_\odot = (98.8 \pm 3.6)^\circ$ and galactic latitude $b_\odot = (-5.9 \pm 3.0)^\circ$ [2] using the following method, modified from the lab handout [1],

$$v_{orbital} = v_\odot \times \cos(b_\odot) \times \cos(l_\odot - l_{object}) + v_r \tag{3}$$

To obtain orbital radius $r_{orbital}$ of an object at galactic longitude $l_{object}$, we used the solar orbital radius $(7.4 \pm 0.3)\,\mathrm{kpc}$ [3] in the following method from the lab handout [1],

$$r_{orbital} = r_\odot \times \sin(l_{object}) \tag{4}$$

We then determine the mass $m$ within each radius $r$ by using the calculated orbital velocity $v$, and reversing the circular orbit equation,

$$m = \frac{v^2 \times r}{G} \tag{5}$$

where $G$ is the gravitational constant, equal to $6.674\,08 \times 10^{-11}\,\mathrm{m^2\,kg^{-1}\,s^{-2}}$ [5].

## 3 Analysis

As was briefly described above in the methodology, there are three main equations which we used to convert the frequency data at each galactic longitude into a rotational curve for the milky way galaxy. These three equations were all derived from the lab handout [1], and the uncertainties determined using the well-known propagation of errors formula,

$$\sigma_f = \sqrt{\sigma_i^2 \times \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2} \tag{6}$$

### 3.1 Radial Velocity

As discussed in the methodology, a cloud emitting at a certain frequency $f_{emit}$ will be observed with a redshifted frequency $f_{obs}$. Using the speed of light $c = 299\,792\,458\,\mathrm{m\,s^{-1}}$ [5], we can use the following equation to calculate the radial velocity of the cloud

$$v_r = \frac{(f_{emit} - f_{obs})}{f_{obs}} \times c \tag{7}$$

The given emitted frequency and the speed of light are taken to be precise values, giving the uncertainty $\sigma_{v_r}$ to be

$$\sigma_{v_r} = \sqrt{\sigma_{f_{obs}}^2 \times \left(\frac{f_{emit}}{f_{obs}^2} \times c\right)^2} \tag{8}$$

Where the uncertainty in the observed frequency value is determined from the "left hand start" method discussed in the methodology.

## 3.2 Orbital Velocity of Each HI Cloud

The next is to determine the true tangential velocity of an object orbiting the Milky Way galaxy at each longitude. To do this, we must consider that the Earth is orbiting around it's axis as it rotates around the Sun, and the Sun rotates around the Milky Way. The data we obtained at each galactic longitude has already accounted for the redshift associated with the Earth's rotation around its axis and its orbit around the sun, which is a product of the time and location where the data was taken.

Our equation, then, must take into account the orbit of the sun around the Milky Way, $v_{\odot} = (299 \pm 15)\,\mathrm{km\,s^{-1}}$ towards a galactic longitude $l_{\odot} = (98.8 \pm 3.6)^{\circ}$ and galactic latitude $b_{\odot} = (-5.9 \pm 3.0)^{\circ}$ [2], which we have constructed around the geometry detailed in the lab handout [1]. It follows that the equation for the orbital velocity $v_{orbital}$ is

$$v_{orbital} = v_{\odot} \times \cos(b_{\odot}) \times \cos(l_{\odot} - l_{object}) + v_r \tag{9}$$

Propagating the error for this value, we get a very long equation for the uncertainty in the orbital velocity, $\sigma v_{orbital}$, to be

$$\begin{aligned} \sigma_{v_{orbital}} = \sqrt{\begin{aligned}&\sigma^2_{v_{\odot}} \times (\cos(b_{\odot})\cos(l_{\odot} - l_{object}))^2 + \\ &\sigma^2_{b_{\odot}} \times (-v_{\odot}\sin(b_{\odot})\cos(l_{\odot} - l_{object}))^2 + \\ &\sigma^2_{l_{\odot}} \times (v_{\odot}\cos(b_{\odot})\sin(l_{\odot} - l_{object}))^2 + \sigma^2_{v_r}\end{aligned}} \end{aligned} \tag{10}$$

## 3.3 Orbital Radius

The final step to constructing the rotation curve is to determine the radius of the orbit of the object at a particular galactic longitude. For this equation, we only need one addition value, the orbital radius of the Sun around the Milky Way's center, $r_{\odot} = (7.4 \pm 0.3)\,\mathrm{kpc}$ [3]. From the lab handout [1],

$$r_{orbital} = r_{\odot} \times \sin(l_{object}) \tag{11}$$

The uncertainty of this value, again following from the standard error propagation formula, is

$$\sigma r_{orb} = \sqrt{\sigma^2_{r_{\odot}} \times (\sin(l_{object}))^2 + \sigma^2_{l_{object}} \times (r_{\odot}\cos(l_{object}))^2} \tag{12}$$

## 3.4 Summarizing the Data and Determining the Mass

Below is a table summarizing the values and their uncertainties we determined from the data, Figure 3a, which shows the rotational curve we were able to derive by plotting the calculated orbital velocity against the calculated orbital radii.

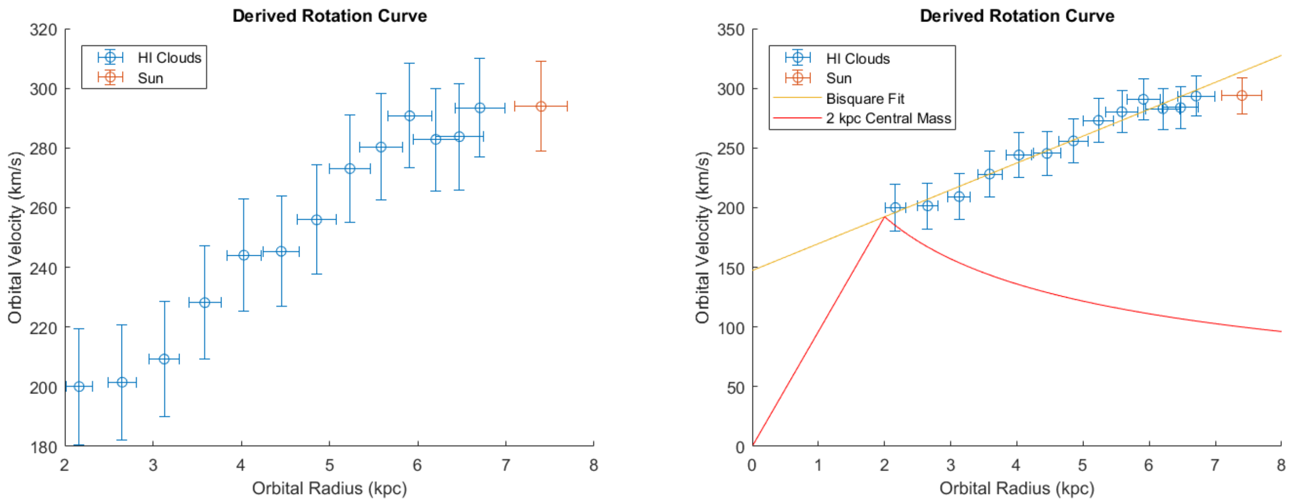
For any radius, $r$, we can determine the mass, $m$, enclosed by this radius using the equation

$$m = \frac{v^2 \times r}{G} \tag{13}$$

Where, again, $v$ is the calculated orbital velocity and G is the gravitational constant $6.674\,08 \times 10^{-11}\,\mathrm{m^2\,kg^{-1}\,s^{-2}}$. [5] We can determine an uncertainty on the enclosed mass by propagation of errors dependent on the uncertainty of our calculated orbital velocity $\sigma_v$, which yields

$$\sigma_m = \sqrt{(\frac{2vr}{G})^2 \sigma_v^2} \tag{14}$$

Figure 3b shows line of best fit, found using Bisquare remainder in *cftool*, as well as the previous centrally dominant galactic model to 2 kpc, scaled to the mass suggested by the linear regression of the observational data. This linear fit suggests that a radius of 4 kpc from the galactic center contains $1.0425 \times 10^{41} \pm 4.7779 \times 10^{9}$kg, or $5.2412 \times 10^{10} \pm 2.4022 \times 10^{-21}\mathrm{M_{\odot}}$.




(a) Orbital velocities against orbital radii, calculated in the manner described above

(b) Line of best fit and rotation curve for centrally dominated galactic model

Figure 3: Orbital velocities and radii of each fully tangential HI cloud, derived from data

| $l_{object}$(rad) | $\sigma_{l_{object}}$(rad) | $v_r$(m s$^{-1}$) | $\sigma_{v_r}$(m s$^{-1}$) | $v_{orbital}$(km s$^{-1}$) | $\sigma_{v_{orbital}}$(km s$^{-1}$) | $r_{orbital}$(kpc) | $\sigma_{r_{orbital}}$(kpc) |
|---|---|---|---|---|---|---|---|
| 0.2967 | 0.01745 | 157610 | 1648.0 | 200030 | 19386 | $6.6767 \times 10^{19}$ | $4.6749 \times 10^{18}$ |
| 0.3665 | 0.01745 | 138640 | 1647.8 | 201490 | 19291 | $8.1838 \times 10^{19}$ | $4.9853 \times 10^{18}$ |
| 0.4363 | 0.01745 | 126280 | 1647.7 | 7209260 | 19160 | $9.6511 \times 10^{19}$ | $5.3251 \times 10^{18}$ |
| 0.5061 | 0.01745 | 125450 | 1647.7 | 228150 | 18997 | $1.1071 \times 10^{20}$ | $5.6831 \times 10^{18}$ |
| 0.5760 | 0.01745 | 122160 | 1647.6 | 244080 | 18802 | $1.2438 \times 10^{20}$ | $6.0496 \times 10^{18}$ |
| 0.6458 | 0.01745 | 104840 | 1647.4 | 245380 | 18579 | $1.3743 \times 10^{20}$ | $6.4168 \times 10^{18}$ |
| 0.7156 | 0.01745 | 97426 | 1647.3 | 255910 | 18331 | $1.4982 \times 10^{20}$ | $6.7779 \times 10^{18}$ |
| 0.7854 | 0.01745 | 97426 | 1647.3 | 273080 | 18062 | $1.6148 \times 10^{20}$ | $7.1273 \times 10^{18}$ |
| 0.8552 | 0.01745 | 88345 | 1647.2 | 280310 | 17776 | $1.7235 \times 10^{20}$ | $7.4604 \times 10^{18}$ |
| 0.9250 | 0.01745 | 83403 | 1647.2 | 290750 | 17479 | $1.8238 \times 10^{20}$ | $7.7731 \times 10^{18}$ |
| 0.9948 | 0.01745 | 61147 | 1646.9 | 282860 | 17175 | $1.9152 \times 10^{20}$ | $8.0622 \times 10^{18}$ |
| 1.0647 | 0.01745 | 48774 | 5763.8 | 283780 | 17752 | $1.9973 \times 10^{20}$ | $8.3246 \times 10^{18}$ |
| 1.1345 | 0.01745 | 46304 | 1646.8 | 293450 | 16572 | $2.0697 \times 10^{20}$ | $8.5580 \times 10^{18}$ |

Table 1: Calculated orbital parameters with uncertainties $\sigma$ for each galactic longitude in the array of $(17 \pm 1)^\circ$ through $(65 \pm 1)^\circ$ in $4^\circ$ intervals, converted to radians

# 4 Discussion

## 4.1 Placing Meaning to Mass Distribution Models

In the first section of this report, we discussed a few theoretical rotation curves. The first is a curve generated by assuming a point source. This curve would be one which could represent our solar system, where most of the mass is concentrated in a central body. A rotational curve of the Solar System would demonstrate that the rotational velocity of a planet orbiting the Sun decreases with the inverse of its orbital radius. The second curve could represent a particle with spin, such as an electron, in which the mass increases linearly with radius from the center. This results in a constant velocity, meaning the entire object spins at the same angular rate no matter which cross section is observed. The final rotational curve we considered is one in which the mass distribution is proportional to the cube of the outward radius. This produces a positive linear relationship between the orbital radius and velocity, and could represent any solid spinning disk.

### 4.2 Special Considerations and Significance of the HI Cloud Spectral Data

One thing that was different in this experiment, as opposed to experiments performed in this class in the past, is the appearance of the spectra we studied. Past experiments used luminosity data over a range of wavelengths, while the data used in this study instead contained frequency and brightness temperature, with highly asymmetric shifts and much more significant baseline noise. Brightness temperature was only used in this study to determine most likely start of signal with uncertainty, in order to find redshifts of the HI clouds with the highest positive radial velocity, which we assumed were also those with fully observed tangential velocity.

### 4.3 Conclusions

The theoretical model of mass distribution discussed in the introduction predicts a linear increase in orbital velocity moving outwards from the galactic center up to 2 kpc and afterwards a steady decrease in orbital velocity inversely proportional to orbital radius. If we were to assume that the distribution of mass in the Milky Way aligns with this model, where the majority of mass is concentrated within 2 kpc and the mass outside of this radius is not present in amounts sufficient to significantly contribute to orbital velocities, then we would expect orbital velocities at radii derived from the data analyzed in this lab to also decrease after 2 kpc.

In contrast to that assumption, however, the velocities and radii derived from the data analyzed in this lab show an increasing linear relationship beyond 2 kpc. This implies that the mass contained within radii greater than 2 kpc does in fact contribute significantly to the orbital velocity of material at those radii, and the assumption therefore does not align with measurements.

## 5 References


[1] Joe DeMartini. Lab 5 handout: Hi rotation curve of the milky way.

[2] J. D. Diaz, S. E. Koposov, M. Irwin, V. Belokurov, and N. W. Evans. Balancing mass and momentum in the Local Group. , 443:1688–1703, September 2014.

[3] Charles Francis and Erik Anderson. Two estimates of the distance to the galactic centre. *Monthly Notices of the Royal Astronomical Society*, 441(2):1105–1114, 2014.

[4] F. J. Lovas, D. R. Johnson, and L. E. Snyder. Recommended rest frequencies for observed interstellar molecular transitions. , 41:451–480, November 1979.

[5] Peter J. Mohr, David B. Newell, and Barry N. Taylor. Codata recommended values of the fundamental physical constants: 2014. *Rev. Mod. Phys.*, 88:035009, Sep 2016.


## 6 Appendix

## Listings



### 6.1 Plotting Consistent Mass Proportion Models

```
% define meters in a kiloparsec
kiloparsec_meters = 1000 * 3.086e+16;

% define mass of the Sun
solar_mass = 1.989e30;
```

```
% define mass of galaxy
galactic_mass = 5.8e11 * solar_mass; % Vayntrub, Alina (2000). "Mass of the Milky Way
    ↪ ". The Physics Factbook. Archived from the original on August 13, 2014.
    ↪ Retrieved May 9, 2007.4

% define total galactic radius
galactic_radius = 15 * kiloparsec_meters;

% get linspace of radii
radii = linspace(0, galactic_radius, 1000);

% start plotting
hold on

% plot constant mass (point mass)
plot(radii / kiloparsec_meters, orbital_velocity(radii, galactic_mass) / 1000);

% plot mass proportional to radius
plot(radii / kiloparsec_meters, orbital_velocity(radii, galactic_mass * (radii /
    ↪ galactic_radius)) / 1000);

% plot mass proportional to radius cubed
plot(radii / kiloparsec_meters, orbital_velocity(radii, galactic_mass * (radii /
    ↪ galactic_radius).^3) / 1000);

% add labels
title('Rotational Velocity vs Orbital Radius: Mass Proportionalities');
xlabel('Orbital Radius (kpc)');
ylabel('Orbital Velocity (km/s)');

% set y axis limits
ylim([0, 1500]);

% add legend
legend('point mass', 'proportional to radius', 'proportional to radius cubed');

hold off
```

### 6.2 Plotting Two-Component Centrally Concentrated Model

```
% define meters in a kiloparsec
kiloparsec_meters = 1000 * 3.086e+16;

% define mass of the Sun
solar_mass = 1.989e30;

% define mass of galaxy
galactic_mass = 5.8e11 * solar_mass;

% get linspace of inner radii
inner_radii = linspace(0, 2 * kiloparsec_meters, 500);

% get linspace of outer radii
outer_radii = linspace(2 * kiloparsec_meters, 8 * kiloparsec_meters, 500);

% start plotting
hold on
```

```
% plot inside inner sphere
plot(inner_radii / kiloparsec_meters, orbital_velocity(inner_radii, galactic_mass * (
    ↪ inner_radii / max(inner_radii)).^3) / 1000, '-b');

% plot outside inner sphere
plot(outer_radii / kiloparsec_meters, orbital_velocity(outer_radii, galactic_mass) /
    ↪ 1000, '-b');

% add labels
title('Rotational Velocity vs Orbital Radius: Spherical Mass of Radius 2 kpc');
xlabel('Orbital Radius (kpc)');
ylabel('Orbital Velocity (km/s)');

hold off
```

### 6.3 Plotting Data from 17° Galactic Longitude

```
% load data at longitude 17
lon17 = readtable('lon17.dat');

% start plotting
hold on

% plot brightness temperature in K vs frequency in mhz
plot(lon17.Var1, lon17.Var2, 'b.');

% add labels
title('Brightness Temperature vs Frequency at 17 deg Galactic Longitude');
xlabel('Frequency (MHz)');
ylabel('Brightness Temperature (K)');

hold off
```

### 6.4 Calculating and Plotting Orbital Velocities and Radii

```
%% calculate orbital parameters

% radial velocities from Doppler shift
radial_velocities = ((HI_emission_frequency - left_hand_start_of_signal_frequencies)
    ↪ ./ left_hand_start_of_signal_frequencies) .* speed_of_light;

% error propagation
radial_velocities_uncertainties = sqrt(...
    left_hand_start_of_signal_uncertainties .^2 .* ((HI_emission_frequency ./
        ↪ left_hand_start_of_signal_frequencies.^2) .* speed_of_light).^2);

% get orbital velocity from line of sight velocity and radial velocity
orbital_velocities = solar_velocity * cos(solar_velocity_galactic_latitude) * ...
    cos(solar_velocity_galactic_longitude - galactic_longitudes) + radial_velocities;

% error propagation
orbital_velocities_uncertainties = sqrt(...
    solar_velocity_uncertainty^2 .* (cos(solar_velocity_galactic_latitude) .* cos(
        ↪ solar_velocity_galactic_longitude - galactic_longitudes)).^2 + ...
```

```
    solar_velocity_galactic_latitude_uncertainty^2 .* (-solar_velocity .* sin(
       ↪ solar_velocity_galactic_latitude) .* cos(solar_velocity_galactic_longitude
       ↪ - galactic_longitudes)).^2 + ...
    solar_velocity_galactic_longitude_uncertainty^2 .* (-solar_velocity .* cos(
       ↪ solar_velocity_galactic_latitude) .* sin(solar_velocity_galactic_longitude
       ↪ - galactic_longitudes)).^2 + ...
    galactic_longitudes_uncertainties^2 .* (solar_velocity .* cos(
       ↪ solar_velocity_galactic_latitude) .* sin(solar_velocity_galactic_longitude
       ↪ - galactic_longitudes)).^2 + ...
    radial_velocities_uncertainties.^2);

% calculate orbital radii using distance from Sun to center of Milky Way
orbital_radii = solar_orbital_radius .* sin(galactic_longitudes);

% error propagation
orbital_radii_uncertainties = sqrt(...
    solar_orbital_radius_uncertainty^2 .* sin(galactic_longitudes).^2 + ...
    galactic_longitudes_uncertainties.^2 .* (solar_orbital_radius .* cos(
       ↪ galactic_longitudes)).^2);

%% Get mass at 2 and 4 kpc

% from cftool with Bisquare remain fit
slope = 7.3e-16;
slope_uncertainty = 1.0300e-16;
y_intercept = 1.473e+05;
y_intercept_uncertainty = 15600;

theoretical_radii = meters_in_kiloparsec:meters_in_kiloparsec:(9 *
   ↪ meters_in_kiloparsec);

masses = (slope * theoretical_radii + y_intercept).^2 .* theoretical_radii /
   ↪ cavendish_constant;
masses_uncertainties = sqrt(...
    slope_uncertainty^2 * (theoretical_radii .* (slope * theoretical_radii +
       ↪ y_intercept)).^2 + ...
    y_intercept_uncertainty^2 * (slope * theoretical_radii + y_intercept).^2);

%% plot rotation curves

% create new figure
figure

% start plotting
hold on;

% plot calculated positions
errorbar(orbital_radii / meters_in_kiloparsec, orbital_velocities / 1000,
   ↪ orbital_velocities_uncertainties / 1000, orbital_velocities_uncertainties /
   ↪ 1000, orbital_radii_uncertainties / meters_in_kiloparsec,
   ↪ orbital_radii_uncertainties / meters_in_kiloparsec, 'o')

% add point for Sun
errorbar(solar_orbital_radius / meters_in_kiloparsec, solar_orbital_velocity / 1000,
   ↪ solar_orbital_velocity_uncertainty / 1000, solar_orbital_velocity_uncertainty /
   ↪  1000, solar_orbital_radius_uncertainty / meters_in_kiloparsec,
   ↪ solar_orbital_radius_uncertainty / meters_in_kiloparsec, 'o')
```

```
plot(0:8, (slope .* ((0*meters_in_kiloparsec):meters_in_kiloparsec:(8*
    ↪ meters_in_kiloparsec)) + y_intercept) / 1000)

% plot theoretical rotation curve
% plot inside inner sphere
plot(inner_radii / meters_in_kiloparsec, orbital_velocity(inner_radii, masses(2) .* (
    ↪ inner_radii / max(inner_radii)).^3) / 1000, '-r');

% plot outside inner sphere
plot(outer_radii / meters_in_kiloparsec, orbital_velocity(outer_radii, masses(2)) /
    ↪ 1000, '-r');

% add labels
title('Derived Rotation Curve');
xlabel('Orbital Radius (kpc)');
ylabel('Orbital Velocity (km/s)');

% add ylimit
ylim([0, 350]);

% add legend
legend('HI Clouds', 'Sun', 'Bisquare Fit', '2 kpc Central Mass', 'Location', '
    ↪ northwest');

hold off;
```

### 6.5 Orbital Velocity Function

```
function [ velocity ] = orbital_velocity( radius, mass )
    %ORBITAL VELOCITY Returns mean orbital velocity given orbital radius and
        ↪ contained mass.

    cavendish_constant = 6.67408e-11;

    velocity = sqrt(cavendish_constant .* mass ./ radius);
end
```