Applying Landscape Evolution to Lunar Water Ice Models - a meta review of literature

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Abstract

Lunar exploration has inherent scientific value, which will be enhanced by eventual human exploration and habitation. Identification of significant areas of lunar water ice could be utilized as a resource to support human habitation, and also serve to further scientific knowledge of the Solar System. Methods exist to find regions with cold-trapped water ice on the Moon; namely, coupling regional illumination modeling with local thermal modeling and remote sensing of water ice deposits with elevation mapping of lunar topography. This paper describes the current state of the field, and enumerates specific challenges and knowledge gaps that could be topics of further research. Notably, in the case of lunar illumination and thermal modeling, this paper highlights incorporation of a dynamically evolving landscape over geologic time to assign an age filter to regions of interest, incorporation of unstructured meshes over traditional raster elevation models, and investigations into discrepancies in reflectance observations compared to other airless bodies.

1 Introduction

Lunar exploration is important for several reasons. Overall, these can be divided into **scientific value** and **human exploration and habitation**.

The 2013 - 2022 Planetary Science and Astrobiology Decadal Survey identified three themes in planetary science: “understanding Solar System beginnings”, “searching for the requirements for life”, and “revealing planetary processes through time” [70]. Similarly, Crawford et al. 2014 [24] details 3 factors of importance to scientific value that can be studied on the lunar surface and subsurface; namely 1) for preservation a more-or-less unchanged geological record of the early Solar System, 2) as a measuring slate on which current activities and processes within the interplanetary inner Solar System (i.e. cosmic rays, solar wind, micrometeorites, and rate of macroscopic impactors) can be measured, and 3) as a controlled and less noisy environment for observational science (i.e. radar astronomy [25]) and experiments in physics [69]. The Moon is a prime candidate for studying all of these topics due to its lack of appreciable atmosphere, lower rate of geologic activity, and distance from Earth-based energetic processes and transmissions. Not only does the Moon show a preserved history of the Solar System and a snapshot of current processes, it also allows several *in-situ* experiments to be made clearly, outside of the much more active geologic and atmospheric environment of the Earth, in fields such as low-energy physics [69] and long-term astrobiology [19].

1.1 Scientific Value

1.2 Human Exploration and Habitation

Lunar exploration is also important for efforts toward crewed exploration and human habitation of extraterrestrial bodies within our star system and beyond. As an airless body, the Moon offers environmental extremes in a location relatively close to Earth. Not only does it offer extreme cold, and regions of surface water ice in permanently-shadowed regions (PSRs), the same topography also offers consistent insolation in near-permanently illuminated regions [41], which is useful for solar power. Additionally, by measure of being tidally locked with Earth, the Moon also provides the possibility for consistent line-of-sight communications between lunar- and Earth-based receivers in areas that are not occluded by topography. To this end, the Lunar Exploration Analysis Group (LEAG), a community based, interdisciplinary advisory council to support NASA lunar exploration objectives, recognized lunar surface illumination as a “strategic knowledge gap” in lunar exploration as of 2012 [50] for finding and mapping permanently shadowed, near-permanently illuminated, and cold-trapped regions on the lunar surface [51].

1.3 Motivation

Water ice is important for lunar exploration, not only for understanding the geological history of the Moon, and by extension the Solar System, but also for human habitation. Water ice is important as 1) a source of propulsion fuel (hydrogen and oxygen) from electrolysis, 2) oxygen for breathing (also from electrolysis), drinking water, and sanitation water, and 3) understanding how water was distributed in the early Solar System. Presence of water ice is used to determine possible landing sites for human and robotic lunar exploration [36].

Prem et al. 2021 [70] enumerates seven key scientific avenues that drive exploration and investigation of water ice; in short, these can be summarized as the **physical properties** of existing lunar volatiles at the poles, the **accumulation rate** of lunar volatiles at the poles, the **transport and loss rates** of lunar volatiles, the **distribution and range** of lunar volatiles beyond the poles, the **transformation cycle** of lunar volatiles, the **history and changes over time** of lunar volatiles, and **whether lunar water is useful** for exploration of the greater Solar System.

As of the writing of this paper, several missions are planned for lunar exploration [2]; NASA alone is planning the Volatiles Investigating Polar Exploration Rover (VIPER) mission, performing *in-situ* measurements in polar craters after 2023 [20], the Lunar Trailblazer smallsat orbiter, adding significant spectral resolution (but small spatial domain) infrared observations for targeted volatile detection and landing site surveying after 2025 [31], and the NASA Artemis program, seeking to send crewed lunar missions, starting in 2025, with the eventual goal of a permanent human presence on the lunar surface and / or in lunar orbit [3].

Flahaut et al. 2020 [36] identified eleven regions of interest (ROIs) around the lunar South Pole with high likelihoods for harboring water ice at the surface or near-surface, and which could be useful landing sites for future crewed and uncrewed exploratory missions. The authors establish previous evidences of lunar water ice, namely “albedo anomalies” [36] in far-ultraviolet (FUV) and near-infrared (NIR) [49], regions of high circular polarization ratio (CPR) in radio [46], and observations of enhanced hydrogen from neutron detection [59]. The authors indicated that landing site suitability mapping can be further narrowed by reducing uncertainties along the rough terrain of the polar region.

2 Selected Topics in Lunar Water Ice

Topics in lunar water ice detection vary across multiple fields, namely planetary science, GIS, remote sensing, and geology. In the connections between the seven scientific avenues highlighted by Prem et al. [70], I have identified several applied topics relevant to my research focus; namely, **illumination modeling**, **thermal modeling**, **volatile / water ice detection**, and **landscape evolution**.

2.1 Illumination Modeling

Illumination and thermal modeling facilitate a regional mapping of areas where 1) topography and 2) obliquity from the orbital axis work together to reduce or eliminate exposure to direct sunlight, reflected sunlight, and starlight. These areas, where the surface is shadowed throughout the orbital period of the body, are called "permanently-shadowed regions" (PSRs). Several PSRs have been modeled and / or observed around the lunar poles, above $\sim 80^\circ$ latitude [17]. Other PSRs have also been modeled at lower latitudes [57].

Contemporary illumination modeling uses elevation data gathered by the Lunar Reconnaissance Orbiter (LRO) and the Chandrayaan series to create a digital elevation model (DEM) of the lunar surface. The model outlined in Mazarico et. al. 2011 [54] extrapolates illumination over a regional DEM by several cycles of lunar nutation (each $\sim 18.6y$ [14]) into the past and future. Illumination sources include direct solar irradiance, directly reflected solar irradiance, starlight, and indirect surface scattering of surrounding topography, based on albedo. Mazarico et al. 2018 [55] describes how sources of illumination are modeled using ephemeris over several cycles of lunar precession, and describes recent improvements to the general model; namely, more flexibility and an increased amount of scattered light bounces along modeled topography.

The general methodology of illumination modeling starts with a global structure capable of creating a view-shed horizon from any arbitrary point (described in Magillo et al. 1994 [52]); while local models exist (such as in Kömle et. al. 2017 [48] for comet 67P / Churyumov-Gerasimenko), most contemporary applications on the lunar surface, such as Barker et al. 2016 [8] and Mazarico et al. 2011 [54], require a global or regional digital elevation model (DEM) in a structured mesh.

Mazarico et. al 2011 [54] and 2018 [55] describe how the illumination model itself is split into two components: view-shed (from the luminance source), and surface reflectance / emissivity to the observer. These implementations perform ray tracing over the DEM from defined luminance sources, calculate absorption and emissivity for the surface material within the defined scatter window, and finally determine reflectance and scattering to the designated regions of interest to provide illuminance.

In application, Kloos et al. 2021 [47] sought to establish the actual amount of photon flux that reaches and emits from lunar PSRs, specifically to determine how effective a remote sensing instrument can find water ice with far-ultraviolet (FUV) or infrared (IR) paired with optical bands. The paper establishes that PSRs are exposed to three main sources of illumination (starlight, Lyman-α photons from the interplanetary medium, and scattered sunlight from surrounding topography). Additionally, it draws from observations made by Paige et al. in 2010 of the LCROSS lunar impact event: that lunar volatiles exist in surface areas at or below $100K$ [65].

For lunar PSRs in the "northern and southern regions", Kloos et al. utilized lidar from the LRO Lunar Orbiter Laser Altimeter (LOLA), all-sky illuminance maps in Lyman-α far-ultraviolet (FUV) [10] and also visible light [58], and surface reflectance data from LRO, SELENE, Clementine, and Chandrayaan-1. The authors found scattered sunlight to be the brightest light to reach PSRs in visible and IR wavelenghts, and that, in FUV, scattered sunlight ties with (and may exceed) light from the interplanetary medium and starlight. Additionally, PSR slopes that faced the equator received less energy than those facing the poles (40% - 60%). The authors concluded that irradiance reaching PSRs is "sufficient to detect water ice" in both FUV and IR, and that IR observations could provide a better signal-to-noise ratio.

While permanently-shadowed regions appear to be the main focus of literature on the subject, Glaser et al. 2018 [41] describes how human habitation would benefit from access to near-permanently **illuminated** regions, for the purposes of solar power and communication with Earth. This elucidates the importance of illumination modeling, not only for detecting permanently-shadowed regions and water ice, but also for maximizing the availability of solar irradiance on-demand.

2.2 Thermal Modeling

PSRs can act as "cold traps" for surface frost (water ice) deposited on the lunar surface. Ahrens et al. 2020 [4] summarizes the state of knowledge of lunar PSRs and illumination, in the context of usage of the LRO Diviner instrument to measure bolometric (radiant *in-situ*) surface temperatures. The paper established that areas on the lunar surface out of direct sunlight have variations in surface temperature dependent upon thermal reflectance and emission from neighboring surfaces [82, 45, 73, 81]. Lunar surface temperatures below $100K$ can sustain water ice [40, 38], and lunar surface temperatures ABOVE $100K$ can NOT sustain water ice [7] because they will "rapidly" sublimate [64]. Additionally, Andersson et al. 2005 shows that water ice on the lunar surface has varying thermal properties between its amorphous ($< 140K$) and crystalline ($> 140K$) forms [6].

The study used bolometric temperature data from LRO Diviner ($500m$) and visible observations from the LROC Wide Angle Camera [15]. The spatial domain of this study included day and night observations of 4 craters (2 on each lunar pole) over 7 years between 2009 and 2016 (10 traverses per crater) [1]. Ahrens et al. concluded that thermal modeling can verify the presence of volatiles in PSRs, given measurements of bolometric temperature.

Although previous observations have been made on lunar craters, the authors detail the knowledge gap in accounting for day - night variation [49, 54]. Specifically, some methodology is needed to determine heat transfer from neighboring surfaces to gather an

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[1] data was acquired from the Planetary Data System Geosciences Node using JMARS - https://jmars.asu.edu/getting-started/lroc-team

understanding of "asymmetric morphologic properties of the craters" and diurnal thermal stability and flux within lunar PSRs [4].

Obliquity and shadowing allow surface frost to accumulate and remain stably (without sublimating) as long as it remains below $100K$ [7]; any surface that spends any amount of time above $100K$ will "rapidly" sublimate [64]. The rates of deposition and sublimation need to be studied further [49, 44]. A line of inquiry started by Watson et al. in 1961 places a constraint on the rate of water ice sublimation from the lunar surface [82], but the rate of deposition is a topic that invites further study, and is one of the seven avenues of scientific inquiry laid out in Prem et al. 2021 [70].

2.3 Volatiles / Water Ice Detection

We know that water ice exists on the lunar surface [39, 40], especially at the poles. Spectroscopic evidence for water ice and hydroxyl includes detections from Deep Impact [80], Chandrayaan-1 [66], the Cassini flyby [18], neutron observations by LPNS [33, 74] and LRO LEND [60, 59], observations of ejecta from the LCROSS event [21], and observations in ultraviolet (UV) [39, 44] and infrared (IR) [49]. Efforts to model transport of volatiles on airless bodies have been ongoing since the 1990s; Butler et al. 1997 [12] details simulationso f randomly-placed volatiles across the surfaces of Mercury and the Moon, and indicates a favorable environment for transporting volatiles from warmer latitudes to polar cold traps [12].

Hayne et al. 2015 [44] shows direct evidence for the existence of water ice within polar PSRs in ultraviolet (UV) and Diviner temperature measurements. This paper establishes a sublimation point for surface water ice of $110K$ on the lunar surface, and $60K$ for CO_2 ice. Additionally, Gladstone et al. 2012 [39] indicates that lunar craters have varying spectral signatures that may be due to shifts in the balance between the accumulation and loss of water ice [39].

Hayne et al. enumerated several informational issues that they sought to solve. To start, the LRO Lyman-α Mapping Project (LAMP) observed some features in ultraviolet that could possibly be water ice; however, the authors noted that 1) these features could also be unusually porous lunar regolith or even CO_2 ice, and 2) the level of mixing between water ice and lunar regolith of the surface is unknown, so any model that fits water ice must also incorporate that possibility. The authors found that features observed by LAMP in ultraviolet closely align with water ice, and that these features are NOT unusually porous lunar regolith. According to the findings, the density of this ice must either be 0.1% - 2.0% by mass OR pure water ice covering up to 10% of the studied regional surfaces. However, Hayne et al. were unable to conclusively *disprove* the possibility of these features actually just being CO_2 ice masquerading as water ice.

Using infrared observations from the Chandrayaan-1 M^3, Lyman-α measurements from LRO LAMP, and lidar-derived altimetry from LRO LOLA, Li et al. 2018 [49] reported that 3.5% of the cold traps studied in the lunar polar regions show definite water ice signatures .

Illumination modeling and water ice detection are not confined to Earth's Moon. Deutsch et al. 2016 [30] describes the existence of water ice on Mercury's poles, cross-validating

Earth-based radar observations of “radar-bright deposits” with imager observations from the MESSENGER spacecraft. It establishes the existence of water ice at Mercury’s poles using imagery from the Mercury Dual Imaging System (MDIS), topography-based DEM derived from the Mercury Laser Altimeter (MLA), and radar reflectance from Earth-based observations. The authors found that 46% of polar PSRs surveyed in imagery, and 43% of PSRs surveyed in lidar, collocated with radar-bright material observed from Earth. “Both approaches produce consistent catalogues of shadowed craters with diameters $>= 6km$ from $80°N$ to the pole” [30]. These findings suggest that these radar-bright regions are volatiles / water ice. Additionally, although not all of the studied PSR candidates were collocated with radar-bright deposits, the occurrence of these specific non-collocated PSRs were observed to increase along Mercury’s longitudinal poles, but not along its hot-cold poles. The authors suggest the possibility that the lack of radar-bright deposits in these regions is due to low visibility angle from Earth-based observatories, and not due to an actual pattern on the surface. Deutsch et al. indicated that further research could be done to investigate commission / omission between imagery and lidar-derived DEMs (such as in Prokofiev crater), improvements to radar observations of Mercury’s polar regions to validate the presence (or lack) of radar-bright deposits in non-collocated regions along the hot-cold poles, and improvements to topographical reconstructions of Mercury’s shallower craters (specifically in the secondary craters around Prokofiev).

Unlike Mercury, radar observations of Earth’s Moon do not reveal obvious radar-bright regions [13, 78] in analogous cold-trapped polar PSRs, even in areas strongly suspected or known to harbor surface frost. This discrepancy could possibly be a further avenue of research, and could be attributed to the “radar-opaque” properties of lunar regolith [81] covering volatile deposits.

Vasavada et al. 1999 [81] compares water ice sublimation rates on the Moon and Mercury, and models a “thin regolith layer” covering deposits in order to co-register radar-bright regions with Lyman-alpha observations. The authors utilized radar observations of Mercury and the Moon [78] to model the properties of water ice on the floors of bowl-shaped craters. The model included direct insolation, thermal and solar scattering, and the thermophysical properties of regolith variable with depth. Additionally, the authors modeled cases where a “thin regolith layer” might cover the volatile deposit, insulating the deposit from direct exposure to insolation and the effects of the near-vacuum. The authors found that water ice deposits on the lunar surface that exist within cold traps are stable (sustaining) within $2°$ of the lunar poles, and have slow sublimation (in the order of billions of years) within $13°$ of the lunar poles. Additionally, the vast majority of Mercury’s cold traps contained radar-bright features and observable water ice deposits, even the ones in warmer latitudes. This indicates that Mercury has “unexpectedly high supply rates” of water ice and other volatiles [81], as transport is an ongoing process along the temperature gradient between warmer and colder cold traps. The Moon’s cold traps, in contrast, are not nearly as occupied by observable water ice nor radar-bright features. The authors speculate that this might be due to differences in obliquity between the Moon and Mercury in their respective histories [81].

In addition to surface frost, lunar volcanic activity also distributes volatiles across the lunar surface. Stern 1999 [79] introduced the possibility that volcanic eruptions could temporary enable a tenuous lunar atmosphere, up to “~10 microbars”, via outgassing of volatiles from ejected internal material. Since 1999, water has been observed in several species of pyroclast, including glass [72], “nominally hydrous” apatite [56], and beads of basalt [61]. Needham et al. 2017 [62] showed how lunar volcanism releases volatiles via outgassing, and further constrained the outgassed atmosphere (put forward by Stern in 1999 [79]) to “a few microbars” depending on the frequency and magnitude of eruptions. Building upon these findings, Aleinov et al. 2021 [5] modeled several scenarios to determine plausible rates of volatile transport and deposition over the geologic history of the lunar environment; namely, perturbations in volcanic activity (both frequency and magnitude of eruptions) to maintain a transient atmosphere, and orbital obliquity of the Moon. While atmospheric pressure is constrained by intermittent volcanic activity along the geologic history of the Moon, lunar orbital obliquity is a larger unknown and possibly could have reached “as high as 50°” [5], which would severely limit the existence and abundance of stable cold trap survival through this period. Aleinov et al. used a global atmospheric circulation model (ROCKE-3D [83]) to model 4 scenarios of volcanic outgassing, focusing on volatile circulation and transport from volcanically-active areas to the poles, to be deposited in existing polar cold traps, over a global structured atmospheric grid (4° × 5° x 40 vertical layers). The modeled start time was $3.5Ga$ (3.5 billion years ago) with corresponding values for solar irradiance and orbital height. The authors used modern topography, which they “assume hasn’t changed since the Late Heavy Bombardment” [5], and 4 scenarios of orbital obliquity. From this model, Alienov et al. found that 1) “thinner atmospheres delivered the water more efficiently”, and 2) obliquity above 25° eliminated all PSRs and prevents cold traps from forming [5] due to a negative volatile flux toward the poles. Further research into this topic could incorporate more accurate estimates of polar wandering [**?**], which would constrain the location of PSRs and their illumination as the spin axis of the Moon shifted.

2.4 Landscape Evolution

Understanding landscape evolution requires an understanding of 1) the rate of macroscopic impactors that induce surface turnover and 2) the rate of space weathering and its effect upon thermal surface properties.

O’Brien et al. 2021 [63] describes an effort to create a comprehensive model of the rate of turnover of lunar regolith. This relates to PSRs as it may change the amount of time that cold traps can hold water ice, as 1) the amount of time regolith is actually exposed to the surface determines the rate of volatile sublimation (via so-called “impact gardening”), and 2) topographical changes over time would invoke changes to illumination and shadowing. O’Brien et al. established the composition and basic properties of lunar regolith [75, 53], and the processes by which we understand space weathering to change the lunar landscape over time [68, 43].

The main issue put forward by O’Brien et al. is a lack of understanding of the *rate* of space weathering, and of specific time frames on lunar regolith. The authors found that “material typically spends only a few million years on the surface”, and that regolith mixing was modeled as reaching “maturity” after “$7Myr$ of cumulative surface exposure”. This suggests that contemporary lunar regolith has only been exposed on the surface for a few million years, indicating a steady rate of mixing and weathering on geologic time scales. The authors call for further research into reducing uncertainties in the known rate of secondary crater production via small impacts, to narrow down the rate of regolith turnover, as well as further research in the rate of iron weathering in regolith exposed on the lunar surface. This research is taken further in Matsumodo et al. 2021 [53], in which the authors study “crystallographic modifications [in iron sulfides,] probably produced by solar wind irradiation” in Apollo-era lunar regolith samples. Additionally, Farrel et al. 2019 [32] provides estimates of water ice sublimation, and Costello et al. 2021 [22] found through modeling that “excavation of material by gardening outpaces burial”, suggesting a net surface-ward movement from impact gardening.

An effective rate of impactors (and thus the rate of regolith turnover and landscape change) can be estimated from a stratigraphy of overlapping crater remnants. This is useful because any appreciable amount of volatiles cold-trapped in a region requires a minimum topographical age. Determining the confluence of these factors depends on the rate of volatile deposition and the observed age of craters in any specific region, which, in combination with an understanding of the rate of volatile transport and deposition, in turn constrains the amount of volatiles that can exist at any cold trap in the present-day.

3 Data and Sensors

Inputs for these studies come from a variety of sensors on several orbiting spacecraft; spacecraft referenced in this paper include the Lunar Reconnaissance Orbiter (LRO) and Chandrayaan-1.

The Lunar Reconnaissance Orbiter (LRO) provides a litany of observational data useful for modeling illumination and detecting water ice. Relevant to studies mentioned here are altimetry and slope characteristic from the **Lunar Orbiter Laser Altimeter (LOLA)** [9, 77], imagery and derived DTM from the **Lunar Reconnaissance Orbiter Camera (LROC)** including the Narrow and Wide Angle Cameras (**NAC**, **WAC**) [71, 1, 9], shadow mapping and bolometric surfaces temperature from the **Diviner Lunar Radiometer Experiment (DLRE)**, hydrogen and volatile detection from the **Lunar Exploration Neutron Detector (LEND)** [57, 15], and radio and polarization observations from **Mini-RF**.

Relevant data mentioned in this paper from Chandrayaan-1 comes from the **Moon Mineralogy Mapper (M^3)**, which provides imagery in multiple bands, including imagery up to $3\mu m$ (NIR), which lies within the tail of IR water absorption [67].

3.1 Limitations

3.1.1 LOLA Thermal Blanket Anomaly

In the case of topography from LOLA (aboard LRO), observations over the day side of the Moon measures $140Hz$ with all 5 receivers in operation. After the spacecraft crosses the polar terminator into night, however, observations are constrained to $80Hz$ - $90Hz$ due to the “thermal blanket anomaly” [76], as the thermal blanket protecting the instrument experiences a change in received solar flux upon crossing the day-night terminator, contracts due to effective temperature change, and pulls the transmitter out of alignment with 3 of the 5 receivers. This results in areas of sparse observations over the poles, where lidar observations have a greater average spacing than in other regions. This effect is compounded at the poles, as the contraction only stabilizes after the orbiter has left the high latitudes in its orbit [77].

3.1.2 Chandrayaan-1 Orbital Anomalies

The Moon Mineralogy Mapper (M^3) aboard Chandrayaan-1 provides $3\mu m$ (NIR) imagery at the far end of its spectral range [42]. However, due to an unexpected anomaly in Chandrayaan-1's star tracker, the spacecraft's attitude and orbit was changed from its design parameters, which led to anomalies in data retrieved by M^3 (due to regular altitude and temperature extremes outside of sensor specifications). This had the effect of "increasing phase, reducing the reflected signal and increasing the effect of surface shadows in the data", as well as increased intervals between observations, which altogether led to a reduced effectiveness of the sensor [11].

Contact with Chandrayaan-1 ended abruptly on August 29th, 2009 [11].

4 Concluding Remarks and Possible Avenues of Research

This paper focuses on the current state of literature of lunar volatile modeling and detection, specifically focusing on identification of probable PSR candidates containing significant amounts of lunar water ice. Several challenges in geospatial science can be addressed to facilitate this effort. The following sections enumerate specific challenges and knowledge gaps that could represent potential areas of focus for my research.

4.1 Geospatial Challenges

Contemporary illumination modeling conventionally acts upon a digital elevation model (DEM) comprised of a structured raster grid [8, 9]. Usage of a traditional DEM raster requires sacrifices to be made in areas with sparse observations (see Section 3.1.1). The unplanned areas of sparse observations resulting from the "LOLA thermal blanket anomaly" cause raster DEMs to have sparse data in a rotating region along the poles and high latitudes, requiring interpolation for continuous area coverage and special consideration in illumination modeling utilizing raster DEMs. The illumination model could thus be improved a DEM built on an unstructured grid, a Triangulated Irregular Network (TIN), or another form of spatial mesh. The technical report by De Floriani et al. [29] describes a preliminary effort to prototype such a model, built on the terrain tree with a multi-resolution model and utilizing level-of-detail (LOD) [29]. The challenges involved in this effort involve building an illumination model over multiple regions of varying resolution and parallelizing ray-tracing over the space of mesh faces [23].

Further implementation of visibility computation over terrain trees [27, 26] could provide the capacity for ingestion of larger datasets into visibility and viewshed applications, and possibly lead into further improvements to illumination modeling. This would also include improvements to ray-tracing and rendering methodologies of spatial data on current hardware, including occlusion and on-demand mesh subsetting of LiDAR-derived unstructured grids [23]. There are also efforts to implement multi-resolution and level-of-detail (LOD) into visibility modeling methods in order to reduce data complexity, in both triangular and tetrahedral meshes [34, 37, 16, 26, 27]. Additionally, an end-product could possibly involve data fusion between a contemporary raster DEM, with enhanced measures of uncertainty [9], and a new unstructured triangle mesh.

4.2 Knowledge Gaps

While illumination modeling supports cold traps and water-ice-candidate mapping, contemporary modeling does not appear to incorporate long-term geologic evolution of the landscape (from impactors and space weathering) and its effect upon illumination topography and the thermal properties of local regoliths. PSRs that currently exist on the lunar surface might potentially have not existed for long enough to have accumulated a significant amount of water ice, depending on the rate of deposition and their topographical age.

Further implementation of unstructured meshes into the illumination modeling process (to replace raster DEMs) [29, 35, 84, 27, 26, 52, 28] could improve the illumination modeling process and provide the capacity for ingestion of larger datasets. This would also include improvements to ray-tracing and rendering methodologies of spatial data on current hardware, including occlusion and on-demand mesh subsetting of lidar-derived unstructured grids [16, 23].

Statistical mapping and geological studies could be employed to investigate reasons behind the lack of lunar radar-bright regions [13, 78] in cold-trapped regions.

The Moon Mineralogy Mapper (M^3) experiences several data anomalies due to issues with the spacecraft star tracker (see Section 3.1.2). Using upcoming *in-situ* data from the VIPER mission [20], as well as small-area infrared imagery from the upcoming Lunar Trailblazer mission [31], Lunar Polar Hydrogen Mapper (LUNAH-MAP), Lunar Ice Cube, and Lunar Flashlight (infrared), this data could be ground-truthed and corroborated with surface temperatures from Diviner [65] to provide wide-area observations of surface hydration across the lunar poles. This could theoretically correct for anomalies and relative inaccuracies in the data.

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References

[1] Casting Light on Permanently Shadowed Regions | Lunar Reconnaissance Orbiter Camera. http://lroc.sese.asu.edu/posts/979.

[2] Global Exploration Map Supplement – August 2020 | ISECG. Technical report, August 2020.

[3] NASA's Lunar Exploration Program Overview, September 2020.

[4] C. J. Ahrens, N. Petro, and M. A. Siegler. Preliminary Surface Thermal Flux Calculations Using LRO Diviner in Lunar Permanently Shadowed Craters. page 1949, March 2020.

[5] I. Aleinov, M. J. Way, K. Tsigaridis, E. T. Wolf, C. Harman, G. Gronoff, and C. W. Hamilton. The role of the transient volcanically-induced lunar atmosphere in transport and deposition of polar volatiles. (2548):2, 2021.

[6] Ove Andersson and Akira Inaba. Thermal conductivity of crystalline and amorphous ices and its implications on amorphization and glassy water. *Physical Chemistry Chemical Physics*, 7(7):1441–1449, 2005.

[7] Edgar L. Andreas. New estimates for the sublimation rate for ice on the Moon. *Icarus*, 186(1):24–30, 2007.

[8] M. K. Barker, E. Mazarico, G. A. Neumann, M. T. Zuber, J. Haruyama, and D. E. Smith. A new lunar digital elevation model from the Lunar Orbiter Laser Altimeter and SELENE Terrain Camera. *Icarus*, 273:346–355, July 2016.

[9] Michael K. Barker, Erwan Mazarico, Gregory A. Neumann, David E. Smith, Maria T. Zuber, and James W. Head. Improved LOLA elevation maps for south pole landing sites: Error estimates and their impact on illumination conditions. *Planetary and Space Science*, 203:105119, September 2021.

[10] J. L. Bertaux, E. Kyrölä, E. Quémerais, R. Pellinen, R. Lallement, W. Schmidt, M. Berthé, E. Dimarellis, J. P. Goutail, C. Taulemesse, C. Bernard, G. Leppelmeier, T. Summanen, H. Hannula, H. Huomo, V. Kehlä, S. Korpela, K. Leppälä, E. Strömmer, J. Torsti, K. Viherkanto, J. F. Hochedez, G. Chretiennot, R. Peyroux, and T. Holzer. SWAN: A Study of Solar Wind Anisotropies on SOHO With Lyman Alpha Sky Mapping. In B. Fleck, V. Domingo, and A. Poland, editors, *The SOHO Mission*, pages 403–439. Springer Netherlands, Dordrecht, 1995.

[11] Joseph W. Boardman, C. M. Pieters, R. O. Green, S. R. Lundeen, P. Varanasi, J. Nettles, N. Petro, P. Isaacson, S. Besse, and L. A. Taylor. Measuring moonlight: An overview of the spatial properties, lunar coverage, selenolocation, and related Level 1B products of the Moon Mineralogy Mapper. *Journal of Geophysical Research: Planets*, 116(E6), 2011.

[12] Bryan J. Butler. The migration of volatiles on the surfaces of Mercury and the Moon. *Journal of Geophysical Research: Planets*, 102(E8):19283–19291, 1997.

[13] Bruce A. Campbell, Donald B. Campbell, J. L. Margot, Rebecca R. Ghent, Michael Nolan, John Chandler, Lynn M. Carter, and Nicholas J. S. Stacy. Focused 70-cm Wavelength Radar Mapping of the Moon. *IEEE Transactions on Geoscience and Remote Sensing*, 45(12):4032–4042, December 2007.

[14] Dario Camuffo. Lunar Influences on Climate. In Cesare Barbieri and Francesca Rampazzi, editors, *Earth-Moon Relationships*, pages 99–113, Dordrecht, 2001. Springer Netherlands.

[15] Gordon Chin, Scott Brylow, Marc Foote, James Garvin, Justin Kasper, John Keller, Maxim Litvak, Igor Mitrofanov, David Paige, Keith Raney, Mark Robinson, Anton Sanin, David Smith, Harlan Spence, Paul Spudis, S. Alan Stern, and Maria Zuber. Lunar Reconnaissance Orbiter Overview: The Instrument Suite and Mission. *Space Science Reviews*, 129(4):391–419, August 2007.

[16] Paolo Cignoni, Leila De Floriani, Paola Magillo, Enrico Puppo, and Roberto Scopigno. TAn2-visualization of large irregular volume datasets. December 2021.

[17] E Cisneros, A K Boyd, H M Brown, A A Awumah, A C Martin, K N Paris, J Leland, R Z Povilaitis, R Hoppe, L M Davis, N M Estes, and M S Robinson. PERMANENTLY SHADOWED REGIONS ATLAS. page 352.

[18] Roger N. Clark. Detection of Adsorbed Water and Hydroxyl on the Moon. *Science*, 326(5952):562–564, October 2009.

[19] Charles S. Cockell. Astrobiology—What Can We Do on the Moon? *Earth, Moon, and Planets*, 107(1):3–10, December 2010.

[20] A. Colaprete, D. Andrews, W. Bluethmann, R. C. Elphic, B. Bussey, J. Trimble, K. Zacny, and J. E. Captain. An Overview of the Volatiles Investigating Polar Exploration Rover (VIPER) Mission. 2019:P34B–03, December 2019.

[21] Anthony Colaprete, Peter Schultz, Jennifer Heldmann, Diane Wooden, Mark Shirley, Kimberly Ennico, Brendan Hermalyn, William Marshall, Antonio Ricco, Richard C. Elphic, David Goldstein, Dustin Summy, Gwendolyn D. Bart, Erik Asphaug, Don Korycansky, David Landis, and Luke Sollitt. Detection of Water in the LCROSS Ejecta Plume. *Science*, 330(6003):463–468, October 2010.

[22] Emily S. Costello, Rebecca R. Ghent, and Paul G. Lucey. Secondary Impact Burial and Excavation Gardening on the Moon and the Depth to Ice in Permanent Shadow. *Journal of Geophysical Research: Planets*, 126(9):e2021JE006933, 2021.

[23] Patrick Cozzi and Kevin Ring. *3D Engine Design for Virtual Globes*. AK Peters/CRC Press, 2011.

[24] Ian A. Crawford and Katherine H. Joy. Lunar exploration: Opening a window into the history and evolution of the inner Solar System. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 372(2024):20130315, September 2014.

[25] Ian A Crawford and John Zarnecki. Astronomy from the Moon. *Astronomy & Geophysics*, 49(2):2.17–2.19, April 2008.

[26] L. De Fioriani, P. Magillo, and E. Puppo. Building and traversing a surface at variable resolution. In *Proceedings. Visualization '97 (Cat. No. 97CB36155)*, pages 103–110„ Phoenix, AZ, USA, 1997. IEEE.

[27] L. De Floriani, P. Magillo, and E. Puppo. Efficient implementation of multi-triangulations. In *Proceedings Visualization '98 (Cat. No.98CB36276)*, pages 43–50„ Research Triangle Park, NC, USA, 1998. IEEE.

[28] Leila De Floriani. A pyramidal data structure for triangle-based surface description. *IEEE Computer Graphics and Applications*, 9(2):67–78, 1989.

[29] Leila De Floriani, Riccardo Fellegara, and Kenneth Weiss. Direct Illumination Modeling of Planetary Surfaces Based on the Terrain Pyramid. Technical report.

[30] Ariel N. Deutsch, Nancy L. Chabot, Erwan Mazarico, Carolyn M. Ernst, James W. Head, Gregory A. Neumann, and Sean C. Solomon. Comparison of areas in shadow from imaging and altimetry in the north polar region of Mercury and implications for polar ice deposits. *Icarus*, 280:158–171, December 2016.

[31] B. L. Ehlmann, R. L. Klima, C. L. Bennett, D. Blaney, N. Bowles, S. Calcutt, J. Dickson, K. Donaldson Hanna, C. S. Edwards, R. Green, M. A. House, A. Klesh, C. Pieters, C. Seybold, D. Thompson, W. Williamson, and J. Wood. Lunar Trailblazer: A Pioneering Smallsat for Lunar Water and Lunar Geology. page 1740, March 2021.

[32] W. M. Farrell, D. M. Hurley, M. J. Poston, P. O. Hayne, J. R. Szalay, and J. L. McLain. The Young Age of the LAMP-observed Frost in Lunar Polar Cold Traps. *Geophysical Research Letters*, 46(15):8680–8688, 2019.

[33] W. C. Feldman, D. J. Lawrence, R. C. Elphic, B. L. Barraclough, S. Maurice, I. Genetay, and A. B. Binder. Polar hydrogen deposits on the Moon. *Journal of Geophysical Research: Planets*, 105(E2):4175–4195, 2000.

[34] Riccardo Fellegara, Federico Iuricich, and Leila De Floriani. Efficient representation and analysis of triangulated terrains. In *Proceedings of the 25th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, SIGSPATIAL '17, pages 1–4, New York, NY, USA, November 2017. Association for Computing Machinery.

[35] Riccardo Fellegara, Kenneth Weiss, and Leila De Floriani. The Stellar tree: A Compact Representation for Simplicial Complexes and Beyond. July 2017.

[36] J. Flahaut, J. Carpenter, J. P. Williams, M. Anand, I. A. Crawford, W. van Westrenen, E. Füri, L. Xiao, and S. Zhao. Regions of interest (ROI) for future exploration missions to the lunar South Pole. *Planetary and Space Science*, 180:104750, January 2020.

[37] Leila Floriani and Paola Magillo. Algorithms for Visibility Computation on Terrains: A Survey. *Environment and Planning B: Planning and Design*, 30(5):709–728, October 2003.

[38] G. Randall Gladstone, Dana M. Hurley, Kurt D. Retherford, Paul D. Feldman, Wayne R. Pryor, Jean-Yves Chaufray, Maarten Versteeg, Thomas K. Greathouse, Andrew J. Steffl, Henry Throop, Joel Wm. Parker, David E. Kaufmann, Anthony F. Egan, Michael W. Davis, David C. Slater, Joey Mukherjee, Paul F. Miles, Amanda R. Hendrix, Anthony Colaprete, and S. Alan Stern. LRO-LAMP Observations of the LCROSS Impact Plume. *Science*, 330(6003):472–476, October 2010.

[39] G. Randall Gladstone, Kurt D. Retherford, Anthony F. Egan, David E. Kaufmann, Paul F. Miles, Joel W. Parker, David Horvath, Paul M. Rojas, Maarten H. Versteeg, and Michael W. Davis. Far-ultraviolet reflectance properties of the Moon's permanently shadowed regions. *Journal of Geophysical Research: Planets*, 117(E12), 2012.

[40] G. Randall Gladstone, S. Alan Stern, Kurt D. Retherford, Ronald K. Black, David C. Slater, Michael W. Davis, Maarten H. Versteeg, Kristian B. Persson, Joel W. Parker, David E. Kaufmann, Anthony F. Egan, Thomas K. Greathouse, Paul D. Feldman, Dana Hurley, Wayne R. Pryor, and Amanda R. Hendrix. LAMP: The Lyman Alpha Mapping Project on NASA's Lunar Reconnaissance Orbiter Mission. *Space Science Reviews*, 150(1-4):161–181, January 2010.

[41] P. Gläser, J. Oberst, G.A. Neumann, E. Mazarico, E.J. Speyerer, and M.S. Robinson. Illumination conditions at the lunar poles: Implications for future exploration. *Planetary and Space Science*, 162:170–178, November 2018.

[42] Robert O. Green, Carle Pieters, Pantazias Mouroulis, and Timothy Koch. Moon Mineralogy Mapper Imaging Spectrometer Science Measurements. In *2008 IEEE Aerospace Conference*, pages 1–5, Big Sky, MT, USA, March 2008. IEEE.

[43] Bruce Hapke. Space weathering from Mercury to the asteroid belt. *Journal of Geophysical Research: Planets*, 106(E5):10039–10073, 2001.

[44] Paul O. Hayne, Amanda Hendrix, Elliot Sefton-Nash, Matthew A. Siegler, Paul G. Lucey, Kurt D. Retherford, Jean-Pierre Williams, Benjamin T. Greenhagen, and David A. Paige. Evidence for exposed water ice in the Moon's south polar regions from Lunar Reconnaissance Orbiter ultraviolet albedo and temperature measurements. *Icarus*, 255:58–69, 2015.

[45] Andrew P. Ingersoll, Tomas Svitek, and Bruce C. Murray. Stability of polar frosts in spherical bowl-shaped craters on the Moon, Mercury, and Mars. *Icarus*, 100(1):40–47, 1992.

[46] Yaqiu Jin and Niutao Liu. CPR Modeling for Mini-RF Data Study on Lunar PSR Region. In *2018 12th International Symposium on Antennas, Propagation and EM Theory (ISAPE)*, pages 1–1, December 2018.

[47] Jacob L. Kloos, John E. Moores, Paul J. Godin, and Ed Cloutis. Illumination conditions within permanently shadowed regions at the lunar poles: Implications for in-situ passive remote sensing. *Acta Astronautica*, 178:432–451, January 2021.

[48] Norbert I. Kömle, Wolfgang Macher, Patrick Tiefenbacher, Günter Kargl, Ivanka Pelivan, Jörg Knollenberg, Tilman Spohn, Laurent Jorda, Claire Capanna, Valentina Lommatsch, Barbara Cozzoni, and Felix Finke. Three-dimensional illumination and thermal model of the Abydos region on comet 67P/Churyumov-Gerasimenko. *Monthly Notices of the Royal Astronomical Society*, 469(Suppl_2):S2–S19, July 2017.

[49] Shuai Li, Paul G. Lucey, Ralph E. Milliken, Paul O. Hayne, Elizabeth Fisher, Jean-Pierre Williams, Dana M. Hurley, and Richard C. Elphic. Direct evidence of surface exposed water ice in the lunar polar regions. *Proceedings of the National Academy of Sciences*, 115(36):8907–8912, September 2018.

[50] Lunar Exploration Analysis Group GAP-Specific Action Team. Strategic Knowledge Gaps for the "Moon First" Human Exploration Scenario, March 12.

[51] Lunar Exploration Roadmap Steering Committee. The Lunar Exploration Roadmap: Exploring the Moon in the 21st Century: Themes, Goals, Objectives, Investigations, and Priorities, 2013, 2013.

[52] Paola Magillo, Leila De Floriani, and X. V. Viale Benedetto. Computing visibility maps on hierarchical terrain models. In *Proc. 2nd ACM Workshop on Advances in GIS, Page*, 1994.

[53] Toru Matsumoto, Takaaki Noguchi, Yu Tobimatsu, Dennis Harries, Falko Langenhorst, Akira Miyake, and Hiroshi Hidaka. Space weathering of iron sulfides in the lunar surface environment. *Geochimica et Cosmochimica Acta*, 299:69–84, April 2021.

[54] E. Mazarico, G.A. Neumann, D.E. Smith, M.T. Zuber, and M.H. Torrence. Illumination conditions of the lunar polar regions using LOLA topography. *Icarus*, 211(2):1066–1081, February 2011.

[55] Erwan Mazarico, Michael K. Barker, and Joseph B. Nicholas. Advanced illumination modeling for data analysis and calibration. Application to the Moon. *Advances in Space Research*, 62(11):3214–3228, December 2018.

[56] Francis M. McCubbin, Andrew Steele, Erik H. Hauri, Hanna Nekvasil, Shigeru Yamashita, and Russell J. Hemley. Nominally hydrous magmatism on the Moon. *Proceedings of the National Academy of Sciences*, 107(25):11223–11228, June 2010.

[57] J. Andrew McGovern, D. Benjamin Bussey, Benjamin T. Greenhagen, David A. Paige, Joshua TS Cahill, and Paul D. Spudis. Mapping and characterization of non-polar permanent shadows on the lunar surface. *Icarus*, 223(1):566–581, 2013.

[58] Axel Mellinger. A Color All-Sky Panorama Image of the Milky Way. *Publications of the Astronomical Society of the Pacific*, 121(885):1180, October 2009.

[59] R. S. Miller, G. Nerurkar, and D. J. Lawrence. Enhanced hydrogen at the lunar poles: New insights from the detection of epithermal and fast neutron signatures. *Journal of Geophysical Research (Planets)*, 117:E11007, November 2012.

[60] Richard S. Miller, David J. Lawrence, and Dana M. Hurley. Identification of surface hydrogen enhancements within the Moon's Shackleton crater. *Icarus*, 233:229–232, May 2014.

[61] Ralph E. Milliken and Shuai Li. Remote detection of widespread indigenous water in lunar pyroclastic deposits. *Nature Geoscience*, 10(8):561–565, August 2017.

[62] Debra H. Needham and David A. Kring. Lunar volcanism produced a transient atmosphere around the ancient Moon. *Earth and Planetary Science Letters*, 478:175–178, November 2017.

[63] P. O'Brien and S. Byrne. Physical and Chemical Evolution of Lunar Mare Regolith. *Journal of Geophysical Research: Planets*, 126(2):e2020JE006634, 2021.

[64] David A. Paige, Matthew A. Siegler, John K. Harmon, Gregory A. Neumann, Erwan M. Mazarico, David E. Smith, Maria T. Zuber, Ellen Harju, Mona L. Delitsky, and Sean C. Solomon. Thermal Stability of Volatiles in the North Polar Region of Mercury. *Science*, 339(6117):300–303, January 2013.

[65] David A. Paige, Matthew A. Siegler, Jo Ann Zhang, Paul O. Hayne, Emily J. Foote, Kristen A. Bennett, Ashwin R. Vasavada, Benjamin T. Greenhagen, John T. Schofield, Daniel J. McCleese, Marc C. Foote, Eric DeJong, Bruce G. Bills, Wayne Hartford, Bruce C. Murray, Carlton C. Allen, Kelly Snook, Laurence A. Soderblom, Simon Calcutt, Fredric W. Taylor, Neil E. Bowles, Joshua L. Bandfield, Richard Elphic, Rebecca Ghent, Timothy D. Glotch, Michael B. Wyatt, and Paul G. Lucey. Diviner Lunar Radiometer Observations of Cold Traps in the Moon's South Polar Region. *Science*, 330(6003):479–482, October 2010.

[66] C. M. Pieters, J. N. Goswami, R. N. Clark, M. Annadurai, J. Boardman, B. Buratti, J.-P. Combe, M. D. Dyar, R. Green, J. W. Head, C. Hibbitts, M. Hicks, P. Isaacson, R. Klima, G. Kramer, S. Kumar, E. Livo, S. Lundeen, E. Malaret, T. McCord, J. Mustard, J. Nettles, N. Petro, C. Runyon, M. Staid, J. Sunshine, L. A. Taylor, S. Tompkins, and P. Varanasi. Character and Spatial Distribution of OH/H2O on the Surface of the Moon Seen by M3 on Chandrayaan-1. *Science*, 326(5952):568–572, October 2009.

[67] Carle M. Pieters, Joseph Boardman, Bonnie Buratti, Alok Chatterjee, Roger Clark, Tom Glavich, Robert Green, James Head, Peter Isaacson, Erick Malaret, Thomas McCord, John Mustard, Noah Petro, Cassandra Runyon, Matthew Staid, Jessica Sunshine, Lawrence Taylor, Stefanie Tompkins, Padma Varanasi, and Mary White. The Moon Mineralogy Mapper (M^3) on Chandrayaan-1. *Current Science*, 96(4):500–505, 2009.

[68] Carle M. Pieters and Sarah K. Noble. Space weathering on airless bodies. *Journal of Geophysical Research: Planets*, 121(10):1865–1884, 2016.

[69] Andrew E. Potter and Thomas L. Wilson. Physics and astrophysics from a lunar base. *Physics and Astrophysics from a Lunar Base*, 202, 1990.

[70] Parvathy Prem, Ákos Kereszturi, Ariel N. Deutsch, Charles A. Hibbitts, Carl A. Schmidt, Cesare Grava, Casey I. Honniball, Craig J. Hardgrove, Carlé M. Pieters, David B. Goldstein, Donald C. Barker, Debra H. Needham, Dana M. Hurley, Erwan Mazarico, Gerardo Dominguez, G. Wesley Patterson, Georgiana Y. Kramer, Julie Brisset, Jeffrey J. Gillis-Davis, Julie L. Mitchell, Jamey R. Szalay, Jasper S. Halekas, James T. Keane, James W. Head, Kathleen E. Mandt, Katharine L. Robinson, Kristen M. Luchsinger, Lizeth O. Magaña, Matthew A. Siegler, Margaret E. Landis, Michael J. Poston, Noah E. Petro, Paul G. Lucey, Rosemary M. Killen, Shuai Li, Shyama Narendranath, Shashwat Shukla, Thomas J. Barrett, Timothy J. Stubbs, Thomas M. Orlando, and William M. Farrell. Lunar Volatiles and Solar System Science. *Bulletin of the AAS*, 53(4), March 2021.

[71] M. S. Robinson, S. M. Brylow, M. Tschimmel, D. Humm, S. J. Lawrence, P. C. Thomas, B. W. Denevi, E. Bowman-Cisneros, J. Zerr, M. A. Ravine, M. A. Caplinger, F. T. Ghaemi, J. A. Schaffner, M. C. Malin, P. Mahanti, A. Bartels, J. Anderson, T. N. Tran, E. M. Eliason, A. S. McEwen, E. Turtle, B. L. Jolliff, and H. Hiesinger. Lunar Reconnaissance Orbiter Camera (LROC) Instrument Overview. *Space Science Reviews*, 150(1-4):81–124, January 2010.

[72] Alberto E. Saal, Erik H. Hauri, Mauro L. Cascio, James A. Van Orman, Malcolm C. Rutherford, and Reid F. Cooper. Volatile content of lunar volcanic glasses and the presence of water in the Moon’s interior. *Nature*, 454(7201):192–195, July 2008.

[73] James R. Salvail and Fraser P. Fanale. Near-surface ice on Mercury and the Moon: A topographic thermal model. *Icarus*, 111(2):441–455, 1994.

[74] A. B. Sanin, I. G. Mitrofanov, M. L. Litvak, B. N. Bakhtin, J. G. Bodnarik, W. V. Boynton, G. Chin, L. G. Evans, K. Harshman, F. Fedosov, D. V. Golovin, A. S. Kozyrev, T. A.

Livengood, A. V. Malakhov, T. P. McClanahan, M. I. Mokrousov, R. D. Starr, R. Z. Sagdeev, V. I. Tret'yakov, and A. A. Vostrukhin. Hydrogen distribution in the lunar polar regions. *Icarus*, 283:20–30, February 2017.

[75] E. M. Shoemaker, R. M. Batson, H. E. Holt, E. C. Morris, J. J. Rennilson, and E. A. Whitaker. Observations of the lunar regolith and the Earth from the television camera on Surveyor 7. *Journal of Geophysical Research*, 74(25):6081–6119, 1969.

[76] David E. Smith, Maria T. Zuber, Gregory A. Neumann, Frank G. Lemoine, Erwan Mazarico, Mark H. Torrence, Jan F. McGarry, David D. Rowlands, James W. Head III, Thomas H. Duxbury, Oded Aharonson, Paul G. Lucey, Mark S. Robinson, Olivier S. Barnouin, John F. Cavanaugh, Xiaoli Sun, Peter Liiva, Dan-dan Mao, James C. Smith, and Arlin E. Bartels. Initial observations from the Lunar Orbiter Laser Altimeter (LOLA). *Geophysical Research Letters*, 37(18), September 2010.

[77] David E. Smith, Maria T. Zuber, Gregory A. Neumann, Erwan Mazarico, Frank G. Lemoine, James W. Head III, Paul G. Lucey, Oded Aharonson, Mark S. Robinson, Xiaoli Sun, Mark H. Torrence, Michael K. Barker, Juergen Oberst, Thomas C. Duxbury, Dandan Mao, Olivier S. Barnouin, Kopal Jha, David D. Rowlands, Sander Goossens, David Baker, Sven Bauer, Philipp Gläser, Myriam Lemelin, Margaret Rosenburg, Michael M. Sori, Jennifer Whitten, and Timothy Mcclanahan. Summary of the results from the lunar orbiter laser altimeter after seven years in lunar orbit. *Icarus*, 283:70–91, February 2017.

[78] N. J. S. Stacy, D. B. Campbell, and P. G. Ford. Arecibo radar mapping of the lunar poles: A search for ice deposits. *Science*, 276(5318):1527–1530, 1997.

[79] S. Alan Stern. The lunar atmosphere: History, status, current problems, and context. *Reviews of Geophysics*, 37(4):453–491, 1999.

[80] Jessica M. Sunshine, Tony L. Farnham, Lori M. Feaga, Olivier Groussin, Frédéric Merlin, Ralph E. Milliken, and Michael F. A'Hearn. Temporal and Spatial Variability of Lunar Hydration As Observed by the Deep Impact Spacecraft. *Science*, 326(5952):565–568, October 2009.

[81] Ashwin R. Vasavada, David A. Paige, and Stephen E. Wood. Near-surface temperatures on Mercury and the Moon and the stability of polar ice deposits. *Icarus*, 141(2):179–193, 1999.

[82] Kenneth Watson, Bruce C. Murray, and Harrison Brown. The behavior of volatiles on the lunar surface. *Journal of Geophysical Research*, 66(9):3033–3045, 1961.

[83] M. J. Way, I. Aleinov, David S. Amundsen, M. A. Chandler, T. L. Clune, A. D. Del Genio, Y. Fujii, M. Kelley, N. Y. Kiang, L. Sohl, and K. Tsigaridis. Resolving Orbital and Climate Keys of Earth and Extraterrestrial Environments with Dynamics (ROCKE-3D) 1.0: A General Circulation Model for Simulating the Climates of Rocky Planets. *The Astrophysical Journal Supplement Series*, 231(1):12, July 2017.

[84] Kenneth Weiss and Leila De Floriani. Sparse terrain pyramids. In *Proceedings of the 16th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems - GIS ’08*, page 1, Irvine, California, 2008. ACM Press.